Calculation exercise: Solutions

Exercise 1

a) $V_o = \frac{1}{1-D} V_{in} = \frac{1}{1-0.6} \cdot 12\,\text{V} = 30\ \text{V}$

b)
$$V_o = DV_{in}$$
$$D = \frac{V_o}{V_{in}} = \frac{35V}{50V} = 0,7$$

Exercise 2

a) MOSFET is voltage-controlled thyristor. MOSFET starts to conduct when there is high enough voltage connected between gate and source. MOSFET stops to conduct when there is low enough voltage connected between gate and source. The switch is controlled through driver-circuit.

b) When the MOSFET is conducting, the diode is not conducting. All the current flows through the MOSFET. Note, that current AC component flows through the capacitor.

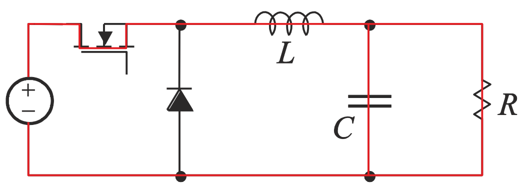


Fig. 1: Current path when MOSFET is conducting

c) The energy stored in the inductor magnetic field resists the current change. When the MOSFET stops to conduct, the inductor current opens the diode to conduct. The circuits needs to be designed on that way, that the inductor current has always current path when the transistor is not conducting.

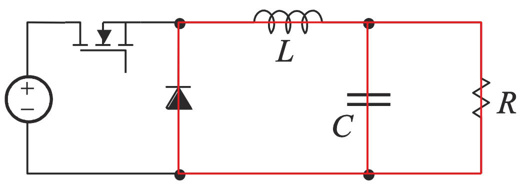
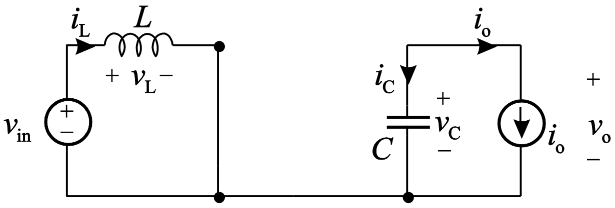


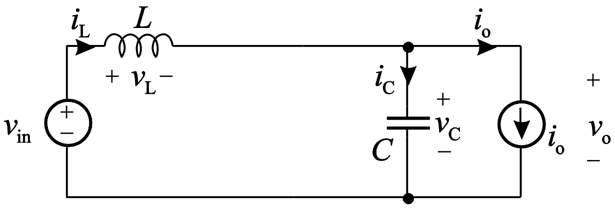
Fig. 2: Current path when MOSFET is not conducting

Exercise 3

a)



Equivalent circuit when the MOSFET conducts



Equivalent circuit when the diode conducts

b)

Inductor current equation when MOSFET conducts:

$$v_{\mathrm{in}} - v_{\mathrm{L\text{-}on}} = 0$$
$$v_{\mathrm{L\text{-}on}} = v_{\mathrm{in}}$$
$$L\frac{\mathrm{d}i_{\mathrm{L\text{-}on}}}{\mathrm{d}t} = v_{\mathrm{in}}$$
$$\frac{\mathrm{d}i_{\mathrm{L\text{-}on}}}{\mathrm{d}t} = \frac{v_{\mathrm{in}}}{L}$$

Capacitor voltage equation when MOSFET conducts:

$$i_{\mathrm{C\text{-}on}} = -i_{\mathrm{o}}$$
$$C\frac{\mathrm{d}u_{\mathrm{C\text{-}on}}}{\mathrm{d}t} = -i_{\mathrm{o}}$$
$$\frac{\mathrm{d}u_{\mathrm{C\text{-}on}}}{\mathrm{d}t} = -\frac{i_{\mathrm{o}}}{C}$$

Inductor current equation when diode conducts:

$$v_{\mathrm{in}} - v_{\mathrm{L\text{-}off}} - v_{\mathrm{C}} = 0$$
$$v_{\mathrm{L\text{-}off}} = v_{\mathrm{in}} - v_{\mathrm{C}}$$
$$L\frac{\mathrm{d}i_{\mathrm{L\text{-}off}}}{\mathrm{d}t} = v_{\mathrm{in}} - v_{\mathrm{C}}$$
$$\frac{\mathrm{d}i_{\mathrm{L\text{-}off}}}{\mathrm{d}t} = \frac{v_{\mathrm{in}} - v_{\mathrm{C}}}{L}$$

Capacitor voltage equation when diode conducts:

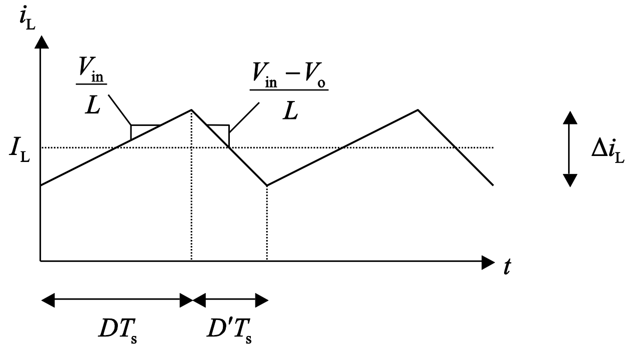
$$i_{\text{C-off}} = i_{\text{L-off}} - i_{\text{o}}$$
$$C\frac{\mathrm{d}u_{\text{C-off}}}{\mathrm{d}t} = i_{\text{L-off}} - i_{\text{o}}$$
$$\frac{\mathrm{d}u_{\text{C-off}}}{\mathrm{d}t} = \frac{i_{\text{L-off}}}{C} - \frac{i_{\text{o}}}{C}$$

These state equations are used to derive the average model of the converter.

c)

The inductor current waveform

- The input and output voltage values are supposed to be constants -> the derivative of inductor current is constant (increasing and decreasing slopes)
- The inductor current ripple is defined as peak-to-peak value in this case. The ripple can be also defined as the peak from the average value. Both methods are used.



Exercise 4

Use the inductor current equation defined in Exercise 3 and the inductor current waveform.

When the MOSFET conducts, the inductor current increase from its minimum value to its maximum value during the time DT_s. The derivative of the inductor current is constant during this time. The maximum allowed inductor current ripple Δi_L is 20 % of the inductor current average value. The inductor current change the amount of Δi_L during the time of DT_s.

The inductor current minimum value $I_L - \frac{\Delta i_L}{2}$

The inductor current maximum value $I_L + \frac{\Delta i_L}{2}$

The slope is known, hence

$$\frac{\Delta y}{\Delta x}=\frac{i_{\text{L-max}}-i_{\text{L-min}}}{\Delta t}=\frac{V_{\text{in}}}{L}$$

$$\frac{\Delta y}{\Delta x}=\frac{\left(I_{\text{L}}+\frac{\Delta i_{\text{L}}}{2}\right)-\left(I_{\text{L}}-\frac{\Delta i_{\text{L}}}{2}\right)}{\Delta t}=\frac{\Delta i_{\text{L}}}{\Delta t}=\frac{V_{\text{in}}}{L}$$

$$\frac{\Delta i_{\text{L}}}{DT_{\text{s}}}=\frac{V_{\text{in}}}{L}$$

The maximum allowed inductor current ripple Δi_{L} is 20 % of the inductor current average value. The required inductance value can be solved

$$\frac{\Delta i_{\text{L}}}{DT_{\text{s}}}=\frac{V_{\text{in}}}{L}\Rightarrow L=\frac{V_{\text{in}}DT_{\text{s}}}{\Delta i_{\text{L}}}=\frac{12\,\text{V}\cdot 0,6\cdot\frac{1}{100\cdot 10^{3}\,\text{Hz}}}{0,2\cdot 0,83\,\text{A}}=434\,\mu\text{H}$$

The equation of the inductor current ripple

$$\Delta i_{\text{L}}=\frac{V_{\text{in}}DT_{\text{s}}}{L}$$

It can be concluded, that the inductor current ripple increases if the inductance value decreases.

Exercise 5

The inductor current waveform of the ideal boost converter is given.
The input voltage is 50 V.

a) The converter operates in CCM, the inductor current is never zero.

b) Duty cycle

$$D=\frac{6,7\mu s}{10\mu s}=0,67$$

c) Input current average value

$$I_{in}=4,5A$$

d) Output voltage

$$V_o=\frac{V_{in}}{D'}=\frac{50V}{1-0,67}\approx 151,5V$$

e) Inductance value

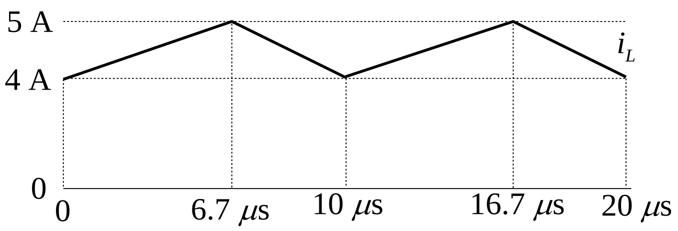
$$\hat{i}_{L-pp}=\frac{V_{in}}{L}\cdot DT_s=\frac{V_{in}-V_o}{L}D'T_s$$

$$\Rightarrow L=\frac{V_{in}DT_s}{\hat{i}_{L-pp}}=\frac{50V\cdot 0,67\cdot 10\mu s}{1A}\approx 335\mu H$$

f) Output current (the converter power losses are not taken into account)

$$P_{in} = V_{in} I_{in} = 50V \cdot 4,5A = 225W$$

$$\Rightarrow I_{out} = \frac{P_{in}}{V_{out}} = \frac{225W}{151,5V} \approx 1,5A$$



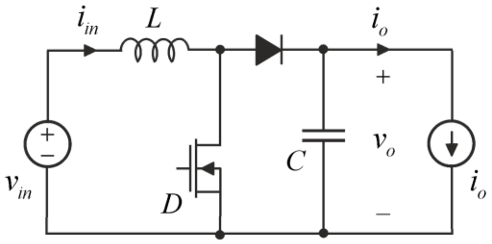


Fig 3. a) Inductor current waveform b) Ideal boost converter

Exercise 6

The above mentioned boost converter is analyzed

a) MOSFET average current

$$I_{mosfet} = I_{in} - I_{diode} = 4,5A - 1,5A = 3A$$

b) MOSFET average voltage

$$V_{mosfet} = V_{out} D' = \frac{V_{in}}{D'} D' = V_{in} = 50V$$

c) Diode average current

$$I_{diode} = \frac{P_{in}}{V_{out}} = \frac{225W}{151,5V} \approx 1,5A$$

d) Diode average voltage

$$V_{diode} = V_{out} D = \frac{V_{in}}{D'} D = \frac{50V}{1-0,67} \cdot 0,67 \approx 101,5V$$

e) Capacitor average voltage

$$V_c = V_o = \frac{V_{in}}{D'} = \frac{50V}{1-0,67} \approx 151,5V$$

Exercise 7

The measured input current of an ideal buck converter is shown in Fig. 4 when the input voltage equals 10 V.

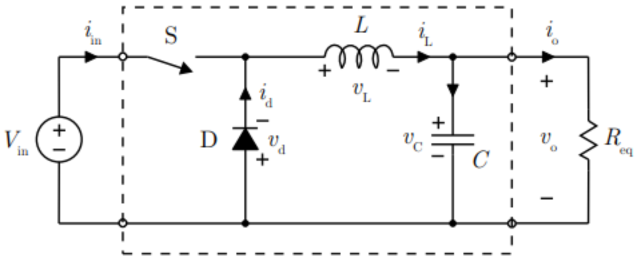


Fig. 4. Buck converter

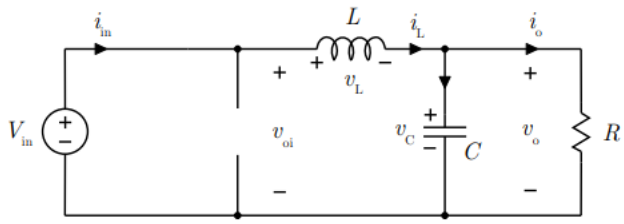


Fig. 5. Buck converter on-time equivalent circuit

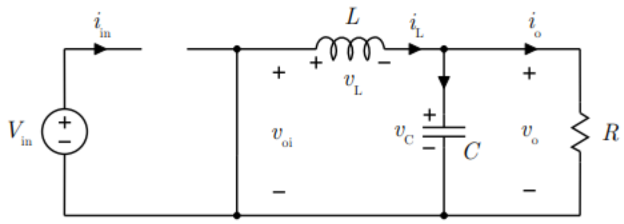


Fig. 6. Buck converter off-time equivalent circuit

Assume that capacitor voltage ripple is negligible. Compute

a) The output voltage

According to the figure, the duty cycle D = 0.5 hence the output voltage is 5 V.

$$D = \frac{t_{on}}{T_s} = \frac{25\mu s}{50\mu s} = 0,5$$

$$V_o = DV_{in} = 0.5 \cdot 10V = 5V$$

b) The switching frequency

$$T_s = \frac{1}{f_s} = 50\mu s \Rightarrow f_s = 20kHz$$

c) The size of the inductor can be combuted based on

$$V_L = L\frac{di}{dt}$$

If the inductor voltage is constant, the derivative term can be presented by the difference in the current and time i.e.

$$V_L = L\frac{\Delta i_L}{\Delta t}$$

Therefore

$$V_{L-on} = L\frac{\Delta i_{L-pp}}{t_{on}} \Rightarrow L = \frac{V_{L-on}t_{on}}{\Delta i_{L-pp}} = \frac{(10V-5V)\cdot 25\mu s}{0.5A} = 250\mu H$$

$$V_{L-off} = L\frac{-\Delta i_{L-pp}}{t_{off}} \Rightarrow L = \frac{V_{L-off}t_{off}}{-\Delta i_{L-pp}} = \frac{(-5V)\cdot 25\mu s}{-0.5A} = 250\mu H$$

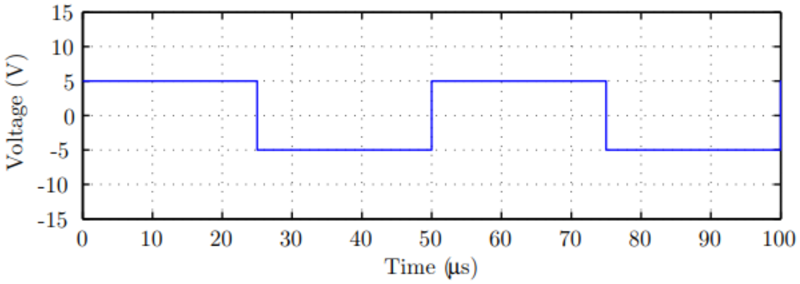


Fig. 7. Inductor voltage waveform

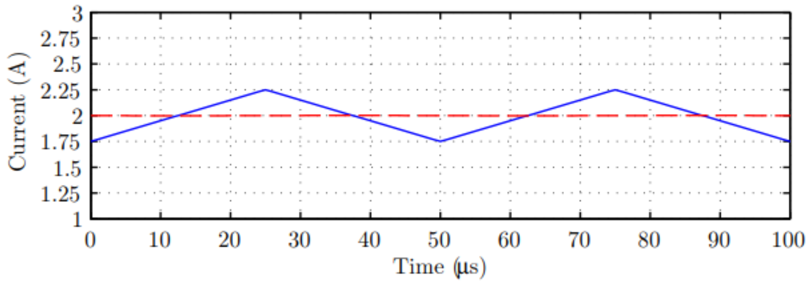


Fig. 7. Inductor current waveform and average current

Exercise 8

In a diode-switched boost (step-up) converter shown in Fig. 3b consider all components to be ideal. Let input voltage be 12 V, output voltage 24 V, switching frequency 20 kHz and inductor L = 260µH. Does the converter operate always in continuous conduction mode (CCM) if output power $P_o \geq 50W$? Assume that capacitor voltage ripple is negligible.

Hint:

$$V_o = \frac{1}{1-D}V_{in}$$

$$v_L = L\frac{di_L}{dt}$$

$$R_{eq} = \frac{V_o}{I_o}$$

Requirement for the CCM operation is

$$I_L > \frac{1}{2}\Delta i_{L-pp}$$

Thus the average inductor current and its peak-to-peak ripple must be computed

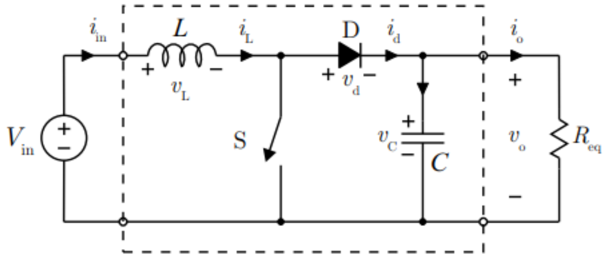


Fig. 8. Power stage of a conventional diode-switched boost (voltage step-up) converter

The duty ratio

$$V_o = \frac{1}{1-D}V_{in} \Rightarrow D = \frac{V_o - V_{in}}{V_o} = \frac{24V - 12V}{24V} = 0.5$$

The average output current

$$I_o = \frac{P_o}{V_o} = \frac{50W}{24V} \approx 2.1A$$

Average inductor current

$$D(-I_o) + (1-D)(I_L - I_o) = 0$$
$$\Rightarrow I_o = (1-D)I_L$$
$$\Rightarrow I_L = \frac{1}{(1-D)}I_o = \frac{1}{(1-0.5)}\cdot 2.1A = 4.2A$$

The peak-to-peak ripple in the inductor current

$$\Delta i_{L-pp} = \frac{V_{L-on}t_{on}}{L} = DT_s\frac{V_{in}}{L} = 0.5\cdot\frac{1}{20kHz}\cdot\frac{12V}{260\mu H} = 1.15A$$

Since

$$I_L > \frac{1}{2}\Delta i_{L-pp}$$
$$4.2A > 0.5\cdot 1.15A$$

The converter operates always in CCM if load power is over 50W. The increase in the output power does not affect to the inductor current ripple but increases the average inductor current.

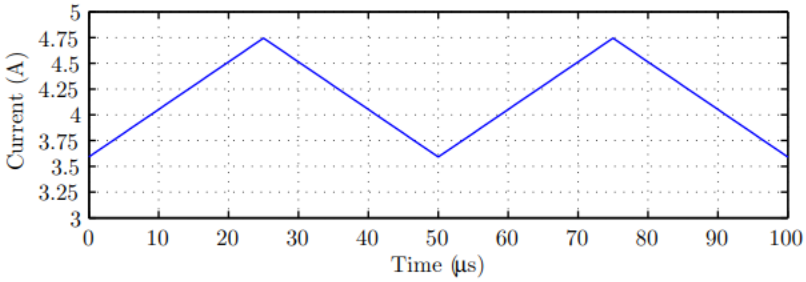
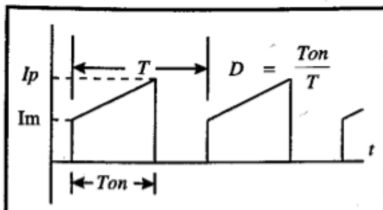


Fig. 9. Inductor current waveform

Exercise 9

Sketch the waveform of the diode voltage and diode current of the ideal boost converter analyzed in Exercise 8 when the output power equals 100 W. Based on the derived current waveform, compute the conduction power loss of the diode. Diode properties are $V_D = 0.7$ V and $r_D = 25\text{m}\Omega$ (assume that the diode behaves as a series connection of a voltage source and a resistor during on-state).

Waveform	Irms	Idc	Iac(rms)
	$\sqrt{D\left(Ip\times Im+\frac{1}{3}(Ip-Im)^2\right)}$	$\frac{D\,(Ip+Im)}{2}$	$\sqrt{Irms^2-Idc^2}$

The duty ratio of the converter may be computed as in the previous Exercise and since the output power equals 100W, the average input and inductor current are now

$$I_L = \frac{1}{(1-D)} I_o = \frac{1}{(1-D)} \frac{P_o}{V_o}$$

$$\Rightarrow I_L = \frac{1}{(1-0.5)} \cdot \frac{100W}{24V} \approx 8.4A$$

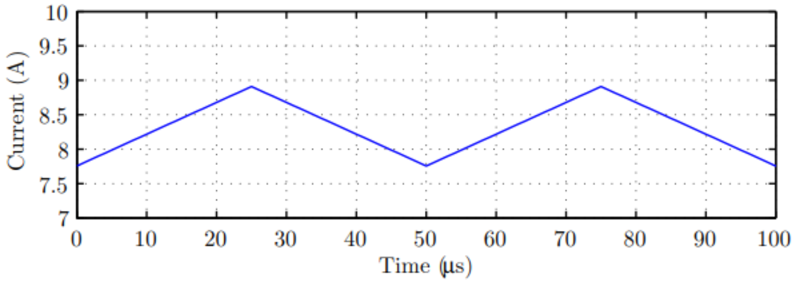


Fig. 10. Inductor current waveform

The power loss of an ideal diode is zero. If a series connection of a voltage source and resistor is used to represent the losses in a diode, the average power loss

$$P_D = V_{D0} I_{D,avg} + r_D I_{D,rms}{}^2$$

The maximum and minimum value of the inductor current are

$$I_{L-\min} = 8.4A - 0.5 \cdot 1.15A \approx 7.8A$$
$$I_{L-\max} = 8.4A + 0.5 \cdot 1.15A \approx 9A$$

The average diode current equals

$$I_{D,avg} = \frac{0.5 \cdot (7.8A + 9A)}{2} \approx 4.2A$$

And the rms-current (see equation table)

$$I_{D,rms} = \sqrt{0.5 \cdot \left(9A \cdot 7.8A + \frac{1}{3}(9A - 7.8A)^2 \right)} \approx 5.9A$$

The average power loss of a diode

$$P_D = V_{D0} I_{D,avg} + r_D I_{D,rms}{}^2 = 0.7V \cdot 4.2A + 25m\Omega \cdot (5.9A)^2 \approx 3.6W$$

The diode voltage and current waveforms are shown. The diode carries the inductor current during off-time of the main switch. The voltage rating of the diode should be at least 24 V i.e. the output voltage with a sufficient margin.

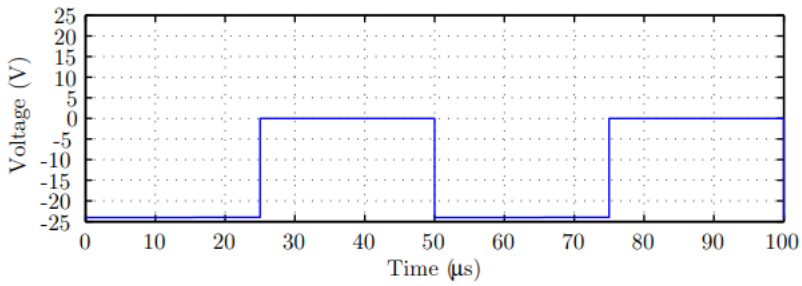


Fig. 11. Diode voltage

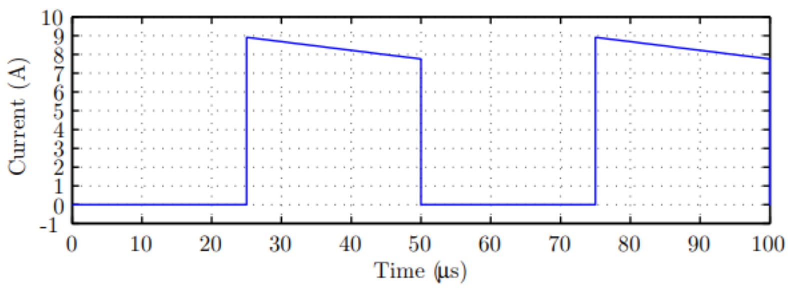


Fig. 12. Diode current