ZNES Zentrum für nachhaltige Energiesysteme
Hochschule | Europa-Universität Flensburg

Introduction to Modelling of Energy Systems

DEEM Training Input Presentation

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Flensburg

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Energy System Modelling, Analysis and Optimization

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Modelling

We Think in Models

Example: "If A happens, B tends to follow, with consequens of C"

Formal Models

Formal models (or mathematical models) are a way to be able to scrutinize and calibrate our models.

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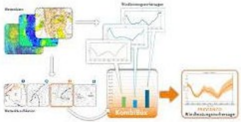
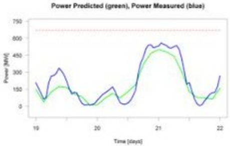
Modelling

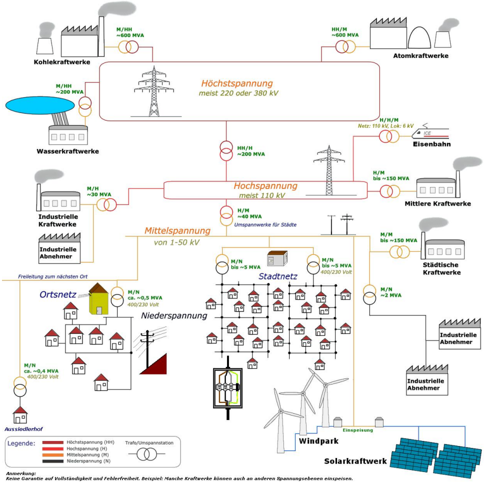
Wind power forecast:





Modelling cont.

Electric grid simulation:



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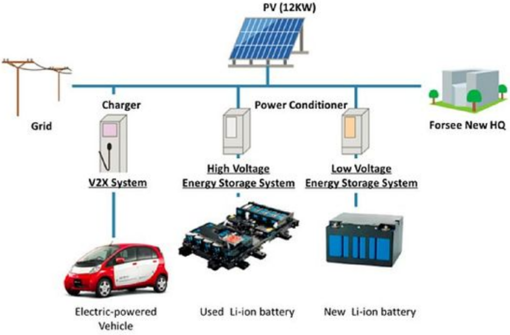
Optimization

System design:



- Minimize Usage from Grid
- Maximize profit / minimize total costs
- Minimize emissions

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Model

In the language use of different scientific disciplines a model is an **object**, that is used by a **subject**. The object is constructed on the basis of a structure or functional analogy to the **original** to solve problems, that can not be done on the original itself.

Characteristics

- Representation
- Shortening, Reduction
- Pragmatism

[Pavlik, 2012]

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Model Purposes

Due to the relation between subject, original and model, the model serves some of the following:

1. Supply of new information about the original.
2. Demonstration and explanation.
3. Reveal properties of the originals, that are not measurable/accesible at the original
4. Optimization of the original.
5. Verification of a hypothesis.
6. Planning of processes for the original.
7. Control of the original.

General Model Types

- Intuitive / conceptual models (also: qualitative or descriptive models)
- Empirical models (e.g econometric models)
- Mechanistic models (physical equations, e.g. Newtons Law)
- System models
  - Input variables (also: referred to as regressor, forcing, exciting, exogenous or independent variables)
  - System structure (internal behavior, physical description, i.e. equations)
  - Output variables (also: response, state, endogenous or dependent variables)

Mathematical Models

... try to capture **crucial parameter** and interactions **of real (nature, technical) objects**, systems or processes **with mathematical relation** and make these computational accessible.

Important steps in modelling process:

1. Formulating of models
2. Evaluation of models
3. Validation of models

[Pavlik, 2012]

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[Pavlik, 2012]

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Explanatory vs. Predictive models

Explaining	Predicting
How a virus affects the body	After you met an infected person you have 50 % risk of being infected
The marginal cost of electricity determines which power plants that are in use	A rainy winter lowers on average the electricity price in summer by 10 %
Electrostatics explains lightning	Given certain weather condition there is a 50 % risk of lightning
Gravitation explain why bodies fall towards earth	G. can predict the velocity of the body after 2 meters in vacuum

Models for Prediction

Four prerequisites that must exist for a model to be validatable:

1. It must be possible to observe and measure the situation being modelled.
2. The situation being modelled must exhibit a constancy of structure in time.
3. The situation being modelled must exhibit constancy across variations in conditions not specified in the model.
4. It must be possible to collect ample data with which to make predictive test of the model.

Prediction, Validation and Energy System Modelling?

Condition 2 and 3 are violated for many energy system (optimization) models[1]*:*

- "Models suggest truth rather than reveal it"
- Model comparison is only a weak benchmark
- Important cognitive function to explore the decision space and observe the interactions among different components
- Result communication becomes a crucial aspect!

[1] Decarolis et al. (2012)

Energy System Models

Energy System Models

can cover every thing from one sector (electricity) and small regions (cities) up to multiple sectors and linked international economies. Models try to answer questions like:

- Which technology will be effective to use?
- How can a energy system with zero CO_2-emissions be designed?
- How prices will develop?
- What grid infrastructure is required?
- How resource scarcity affect the energy system?
- How policies may effect the energy supply?
- How different technologies may interact?

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System Boundaries: What to Exclude? What approach? I

The important thing about models is what they do not include. Depeding on your research question you need to decide what can be excluded to which degree:

- Spatial coverage and resolution?
- Temporal resolution and time-horizon?
- Sectoral coverage?
- Technologies and technology interaction?
- Demand modelling?
- Degree of endogonization?

Regarding method and approaches:

System Boundaries: What to Exclude? What approach? II

- Analytical approach (Top-Down, Bottom Up, Hybrid)
- Underlying method (Optimization, Simulation)
- Mathematical approach (Linear or Mixed-Integer Linear Programming, Agent Based Modelling, other Meta-Heuristics)
- Data requirements

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Classification of Energy System Models I

- A **simulation** model simulates the operation of a given energy-system to supply a given set of energy demands. For example a model is operated in hourly time-steps over a one-year time-period.
- A **scenario** tool usually combines a series of years into a long-term scenario. Typically scenario tools function in time-steps of 1 year and combine such annual results into a scenario of typically 20–50 years.
- **Operation optimization** tools optimize the operation of a given energy-system. Typically these tools are also simulation tools optimizing the operation of a given system.

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Classification of Energy System Models II

- **Investment optimization** tools optimize the investments in an energy-system. Typically investment optimization tools are also scenario tools optimizing investments in new energy stations and technologies.
- An **equilibrium** tool seeks to explain the behaviour of supply, demand, and prices in a whole economy or part of an economy (general or partial) with several or many markets. It is often assumed that agents are price takers and that equilibrium can be identified.

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Types of Energy System Models[2]

Top Down	Bottom Up
use an "economic approach"	use an "engineering approach"
pessimistic estimates on "best" performance	optimistic estimates on "best" performance
explicitly represent technologies	detailed description of technologies
reflect available technologies adopted by the market	reflect technical potential the "most efficient" technologies are given by the
production frontier (which is set by market behavior)	efficient technologies can lie beyond the economic production frontier suggested by market behavior
use aggregated data for predicting purposes	use disaggregated data for exploring purposes
are based on observed market behavior	are independent of observed market behavior
disregard the technically most efficient technologies available, thus underestimate potential for efficiency improvements	disregard market thresholds (hidden costs and other constraints), thus overestimate the potential for efficiency improvements
determine energy demand through aggregate economic indices (GNP, price elasticities), but vary in addressing energy supply	represent supply technologies in detail using disaggregated data, but vary in addressing energy consumption
endogenize behavioral relationships	assess costs of technological options directly
assumes there are no discontinuities in historical trends	assumes interactions between energy sector and other sectors is negligible

[2] Beeck (1999)

Energy Model Families[3]

Model Family	Examples	Primary Focus
Energy system optimization models	MARKAL, TIMES, MESSAGE, OSeMOSYS	Normative scenarios
Energy system simulation models	LEAP, NEMS, PRIMES	Forecasts, predictions
Power systems and electricity market models	WASP, PLEXOS, ELMOD, EMCAS	Operational decisions, business planning
Qualitative and mixed-methods scenarios	DECC 2050 pathways, Stabilization wedges	Narrative scenarios

[3] Pfenninger et al. (2014)

Potential Functions of an Energy System Model

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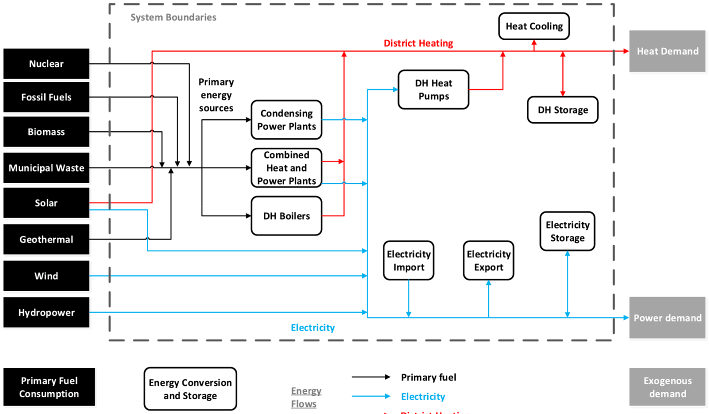
Example: The Balmoral Model [4]



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[4] Wiese et. al (2018)

Constraints for Energy System Models[5]

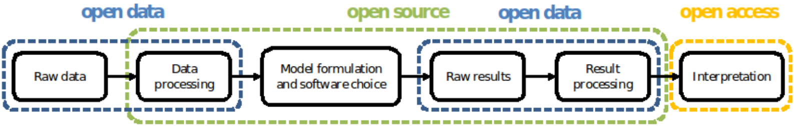
When developing models, decisions must be made as to the mathematical constraints. For example, these might include:

- Energy demand must be satisfied at all times
- User defined CO_2 emissions targets are met.
- Production and consumption of energy must balance, allowing for transmission, storage and losses
- Energy system must satisfy certain other constraints (e.g. suffi- cient capacity to meet peak demand for electricity and heat, capacity to sustain generation during sustained periods of low variational generation)
- Technological links can be specified by users and respected by the model (e.g. enough suitable rooftops for roof-mounted solar PV installations)

[5] Hall et al. (2016)

Open Source Modelling and Open Science

Open modelling



The open modelling process[6] tries to make

- raw, result and processed data,
- scripts for processing data,
- model source code and documentation,
- and publication(s)

accessible and reusable by appropriate licensing.

[6] Pfenninger et al. (2018)

What means open?

Open definition by OKFN

"Open means **anyone** *can* **freely access, use, modify, and share** *for* **any purpose**.*"*[7]

Open Science

"Open Science covers the **strategies and processes** *needed to make* **all** *aspects of the* **scientific process openly accessible and re-usable** *by making use of the potential offered by the digital age."*[8]

Associated terms: Open access, open source, open data, open review, open metrics

[7] https://opendefinition.org/

[8] okfn.de/en/themen/offene-wissenschaft/

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Good reasons for open science

- Ensure scientific standards (transparency, reproducibility etc.)
- Maximize distribution of scientific work and accelerate and optimize usage
- Increased resource efficiency (in particular secondary analysis based on collected data)
- Improve quality due to broader review of model source code, input data
- If public money is used for funding, results should be publicly available
- Increase public acceptance and participation
- Strengthening of author rights through open licenses
- Increased readership and importance of scientific literature

Open source Energy System Analysis Tools

For building bottom-up models:

- PyPSA https://pypsa.org/ (load-flow)
- oemof https://www.oemof.org (multi-sector)
- calliope https://www.callio.pe/
- urbs https://github.com/tum-ens/urbs
- OSeMOSYS http://www.osemosys.org/ (multi-period expansion)
- EnergyPlan (free but **not open source!**)

For calculating wind and pv feedin:

- windpowerlib
- pvlib

For more information checkout https://wiki.openmod-initiative.org/wiki. There is also a mailing list that you can join and ask questions.

Exercise: Modelling

Discuss in groups of 2-3 people

- Think of a model that can be used to help answering your research question.
- What are possible exogenous variables (parameters)?
- What are possible endogenous variables?
- What are constraints of that model?

Scenario Analysis

What Is a Scenario and What Are its Values?

What is a scenario?

A scenario is a construct of language based on 'possibility-statements',

What are Scenarios Good for?

- Show (desired / undesired) effects of certain developments / events
- Provide a basis for discussion
- Help to evaluate alternative options
- Integrate knowledge from various disciplines
- Identification of robust developments,by identifying common elements between scenarios)

Characteristics of Energy System Scenarios

- Description of possible future developments based on today's knowledge
- As knowledge changes, new developments in scenarios may be possible / not possible any longer
- Scenarios are neither deterministic nor stochastic prognosis
- Due to the size and high-dimensional characteristic of energy systems a scenario only provides insight into a small subset (one specific path) of all possible developments / paths.
- Conditional statements are important foundations of scenarios to describe and analyze interdependencies within a system (also for drawn conclusions).
- It's important to have an idea about the uncertainty of conditional (if-then) statements

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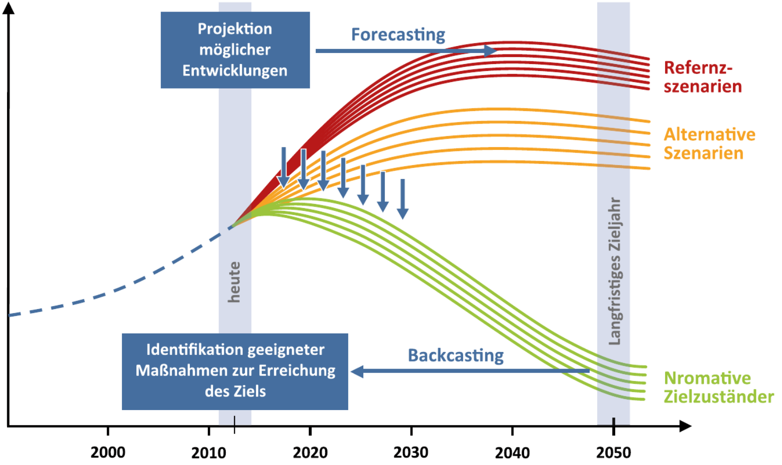
Challenges of Energy Scenario Analysis

- Identification and selection the 'most important' scenarios: 1. exploratory (open and unbiased from the present) or 2. normative (definition of goals)
- Scenarios of complex systems are generally calculated based on computer models (therefore a scenario consist of exogenous, endogenous parameters and the model)
- The model needs to be an adequate representation of the real system
- All exogenous parameters need to be set consistently within each scenario

Forecasting vs. Back-casting



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Import Factors for Future Energy Systems

- decentralized renewable energy (generation),
- centralized renewable energy (generation),
- nuclear energy (generation),
- fossil fuel energy (generation),
- centralized storage (generation and demand),
- electricity exchanges outside country (exchanges),
- transnational initiatives inside country (exchanges)
- GDP and population (demand),
- demand according to new uses (demand),
- energy efficiency (demand)

Modelling Renewable Energy

Goal: Renewable Feedin with high temporal and spatial resolution

- Capacity potential (area)
- Energy potential (wind-speed, solar radiation)
- Models for calulation hourly run of river, solar and wind based on weather data and GIS information

Modelling Demand

Goal: Data with high temporal (hourly) and spatial resolution

- Yearly energy (electricity) demand and hourly demand profiles GIS
- Temporal resolution: for profiles historic data may be used as a starting point
- Top-Down vs. Bottom up
- Statistics to be used for regional resolution:
  - population density
  - industry activity,
  - GDP (maybe also from GDP projection model)

Mathematical definitions and notations

Sets

- Think of as "collections"
- Numbers, objects, etc.
- Notation: $M = \{\text{“}green'', \text{“}yellow'', \text{“}blue''\}$
- $N = \{1, 4, 8, 12\}$
- Use in Linear Programming for sums or multiple constraints

Notations for Sets

Say	Notation
x is in X	$x \in X$
x is not in X	$x \notin X$
Set of all x such that x is real and x is greater than 5	$\{x : x \in \mathrm{R}, x > 5\}$
Every element of Y is a element of X	$Y \subset X$
All elements of a X that are not elements in Y	$X \setminus Y$
For all elements x in X	$\forall x \in X$
f is a function from a set X to a set Y	$f : X \rightarrow Y$

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Examples of Sets

Example 1:

$$M = \{1, 2, 3\}$$

$$x_m, m \in M$$

$$\sum_{m \in M} x_m = x_1 + x_2 + x_3$$

Example 2:

$$x_m - 2 \leq b_m, \qquad \forall m \in M$$

$$\Leftrightarrow$$

$$x_1 - 2 \leq b_1$$

$$x_2 - 2 \leq b_2$$

$$x_3 - 2 \leq b_3$$

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Exercise II

Sets:

$$Y = \{A, B, C\}, \quad K = \{12, 22, 23\}$$

Variable:

$$x_{k,y},\ y \in Y,\ k \in K$$

What is equal to the following expressions:

$$\sum_{y \in Y} x_{k,y} = 10, \qquad \forall k \in K$$

$$x_{k,y} \leq -10, \qquad \forall k \in K,\ \forall y \in Y$$

?

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Energy System Models

Potential Functions of an Energy System Model

1. **Prescribe** – How should the system look like?
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To build a good model you need to know the system. A good way of getting to know the system is to build a model.

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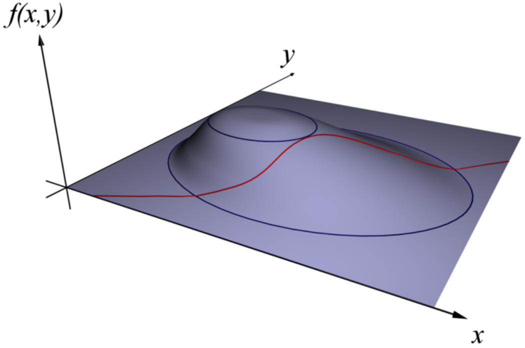
Optimization

Graphical Illustration



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Solving Optimization Problems

Steps

- ↪ Existence of a real world problem
- ↪ Identification as an optimization problem
- ↪ Mathematical formulation (e.g. MILP)
- ↪ Algorithms for solving the model
- ↪ Running the algorithm on a machine
- ↪ Validating the solution
- ↪ Implementation of the solution

Mathematical Optimization

Standard Optimization Problem

$$\text{Minimize} \quad f(x) \tag{1}$$

$$\text{subject to:} \quad g_i(x) \leq 0, \quad i = 1, \ldots, k \tag{2}$$

Important Objects in Mathematical Optimization

- *Decision variables (endogenous variables)*
- *Constraints* (Modelling of restrictions such as capacity constraints)
- *Objective function* Evaluation of the solution (e.g. Cost minimization)
- *Parameters* (exogenous variables) Input values known (e.g. costs)

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General Form

$$\begin{aligned}
\underset{P}{\text{Minimize}} \quad & f(x) && \text{Objective function} \\
\text{subject to:} \quad & g_i(x) \leq 0, \quad i \in I && \text{Inequality constraints} \\
& h_j(x) = 0, \quad j \in J && \text{Equality constraints} \\
& x \in \Omega && \text{Variables}
\end{aligned}$$

We generally have: $\Omega \subseteq \mathrm{R}^n$
$f, g_i, h_j : \mathrm{R}^n \to \mathrm{R}$

Feasible Set and Optimality

$\underset{P}{\text{Minimize}}$	$f(x)$		Objective function
subject to:	$g_i(x) \leq 0,$	$i \in I$	Inequality constraints
	$h_j(x) = 0,$	$j \in J$	Equality constraints
	$x \in \Omega$		Variables

- Feasible set:
  $\mathcal{X}(P) := \{x \in \Omega : g_i(x) \leq 0, h_j(x) = 0, j \in J, k \in K\}$
- If: $\mathcal{X}(P) = \{\emptyset\}$, the problem (P) is called infeasible.
- Optimal solution of P: $x^* \in \mathcal{X}(P)$ s.t.
  $f(x^*) = min\{f(x) | x \in \mathcal{X}(P)\}$
- If $f(x^*) = -\infty$, the problem is unbounded.

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Constraint and Objective Types in Optimization

Depending on the relation between decision (dependent) variables, constraints can be classified in three categories:

Equation type	Example
linear	$c \cdot x_1 + x_2 \cdot b \leq 10x_3$
non-linear	$x_1 \cdot sin(x) = 0$
quadratic	$c_1 \cdot x_1 \cdot x_2$ (in objective)

Where x_i are the decision variables.

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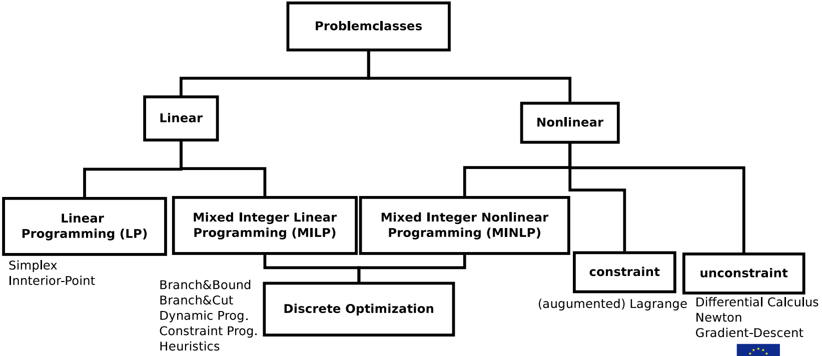
Variable Types in Optimization

Variable-type	Properties
Continuous	$x_c \in 0 \leq X^- \leq x_c \leq X^+$
Free	non-positive possible
Discrete	$x_d \in \{0, 14, 2.4, \dots\}, X^- \leq x_d \leq X^+$
Integer	$x_z \in \{0, 1, 2, \dots\}, X^- \leq x_z \leq X^+$
Binary	$x_b \in \{0, 1\}$
Semi-continuous	$x_{sc} \in \{0 \ \vee X^- \leq x_{sc} \leq X^+\}$

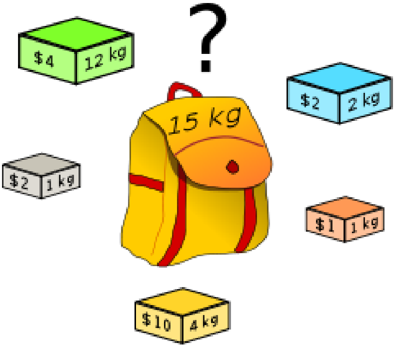
Classification and Types of Optimization problems

- What is the domain of the decision variables ?
- What relation exists between the variables inside an equation?



The Knap-Sack-Problem

Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.



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Mathematical Formulation of the Knapsack-Problem

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{n} v_i x_i \\ \text{subject to:} \quad & \sum_{i=1}^{n} w_i x_i \leq W \\ & x_i \in \{0, 1\} \end{aligned}$$

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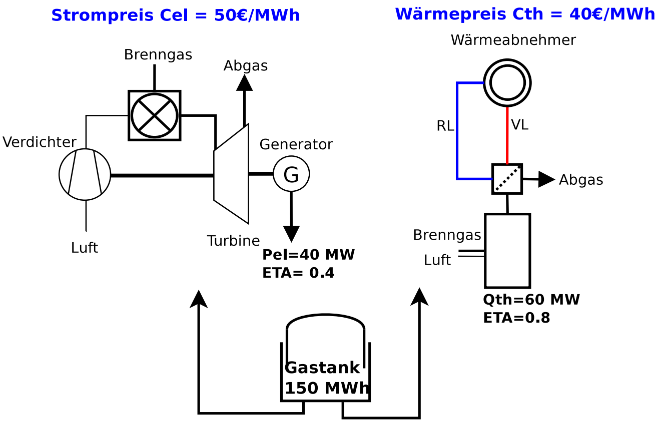
Linear Programming

Modelling Process in Linear Programming

- Sketch / Describe a model
- Develop an abstract model (LaTeX)
- Implement the model (Algebraic Modelling Language)
- Populate model with data
- Solve the model (solver)
- Analysis of results

Simple Energy Optimization Problem



What is the cost optimal way to use the gas?

Recap: Mathematical optimization

Standard optimization problem

$$\text{Minimize} \quad f(x) \tag{3}$$

$$\text{subject to:} \quad g_i(x) \leq 0, \quad i = 1, \ldots, k \tag{4}$$

$$h_j(x) = 0, \quad j = 1, \ldots, l \tag{5}$$

Elements to identify:

- Decision variables
- Objective function
- Constraints

Definition of a Linear Program: Matrix-Notation

Linear Program in matrix-standard form

$$\begin{aligned} \text{min} \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$$c \in \mathbb{R}^n, \; A \in \mathbb{R}^{mxn}, \; b \in \mathbb{R}^m$$

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Linear Programs

Objective:

$$\text{min:} \quad c_1x_1 + \ldots + c_nx_n$$

Constraints

subject to:

$$\begin{array}{cccc} a_{11}x_1 & + \ldots & +a_{1n}x_n \leq & b_1 \\ a_{21}x_1 & + \ldots & +a_{2n}x_n \leq & b_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}x_1 & + \ldots & +a_{mn}x_n \leq & b_m \\ & x_i \geq 0, & i = 1 \ldots n & \end{array}$$

m constraints and n variables

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Linear Programs

Objective:

$$\text{min:} \quad c_1x_1 + \ldots + c_nx_n$$

Constraints

subject to:

$$\begin{array}{ccccc} a_{11}x_1 & + \ldots & +a_{1n}x_n & \leq & b_1 \\ a_{21}x_1 & + \ldots & +a_{2n}x_n & \leq & b_2 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1}x_1 & + \ldots & +a_{mn}x_n & \leq & b_m \\ \end{array}$$

$$x_i \geq 0, \quad i = 1 \ldots n$$

m constraints and n variables

Constraints in Matrix-Notation

$$
\begin{array}{llll}
a_{11}x_1 & +\dots & +a_{1n}x_n \leq & b_1 \\
a_{21}x_1 & +\dots & +a_{2n}x_n \leq & b_2 \\
\vdots & \vdots & \vdots & \vdots \\
a_{m1}x_1 & +\dots & +a_{mn}x_n \leq & b_m
\end{array}
$$

$$\Downarrow$$

$$
\underbrace{\begin{pmatrix} a_{1,1} & a_{2,1} & \dots & a_{m,1} \\ a_{1,2} & a_{2,2} & \dots & \\ \vdots & & \ddots & \vdots \\ a_{1,n} & \dots & & a_{m,n} \end{pmatrix}}_{=A}
\underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}}_{=x\ (Vars)}
\leq
\underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_{=b\ (RHS)}
$$

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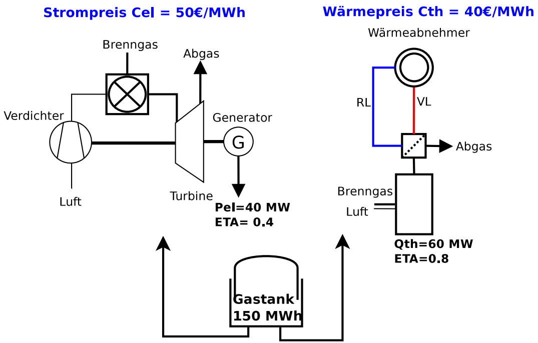
Constraints in Matrix-Notation

$$\begin{array}{llll} a_{11}x_1 & +\dots & +a_{1n}x_n \leq & b_1 \\ a_{21}x_1 & +\dots & +a_{2n}x_n \leq & b_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}x_1 & +\dots & +a_{mn}x_n \leq & b_m \end{array}$$

$$\Downarrow$$

$$\underbrace{\begin{pmatrix} a_{1,1} & a_{2,1} & \dots & a_{m,1} \\ a_{1,2} & a_{2,2} & \dots & \\ \vdots & & \ddots & \vdots \\ a_{1,n} & \dots & & a_{m,n} \end{pmatrix}}_{=A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}}_{=x\ (Vars)} \leq \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_{=b\ (RHS)}$$

Exercise: Optimization model (1 Timestep e.g. MWh)



- Decision Variables? Parameters?
- Objective Function?
- Constraints?

Simple energy-optimization model cont'd

Input-Data

$C_{th} = 40$ Euro/MWh, $C_{el} = 50$ Euro/MWh
$P_{max} = 40$ MW, $Q_{max} = 60$ MW, $F_{max} = 150$ MWh
$\eta_{P_{el}} = 0.4, \ \eta_{Q_{th}} = 0.8$

Linear Program

$$\begin{aligned} \text{minimize:} \quad & -40Q - 50P \\ \text{subject to:} \quad & 1.25Q + 2.5P \leq 150 \\ & Q \leq 60 \\ & P \leq 40 \\ & P, Q \geq 0 \end{aligned}$$

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Simple energy-optimization model cont'd

Input-Data

$C_{th} = 40$ Euro/MWh, $C_{el} = 50$ Euro/MWh

$P_{max} = 40$ MW, $Q_{max} = 60$ MW, $F_{max} = 150$ MWh

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$$
\begin{aligned}
\text{minimize:} \quad & -40Q - 50P \\
\text{subject to:} \quad & 1.25Q + 2.5P \leq 150 \\
& Q \leq 60 \\
& P \leq 40 \\
& P, Q \geq 0
\end{aligned}
$$

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Graphical representation of LPs

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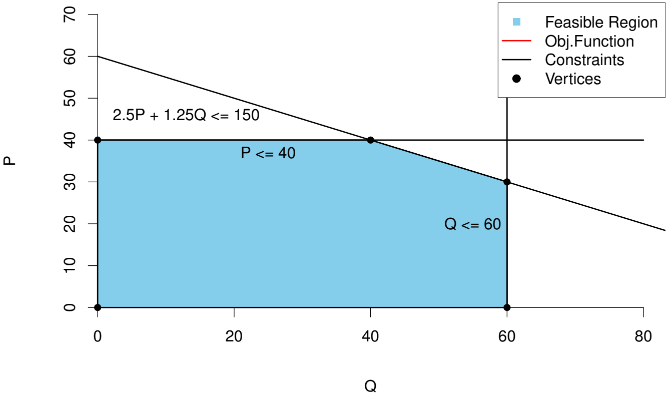
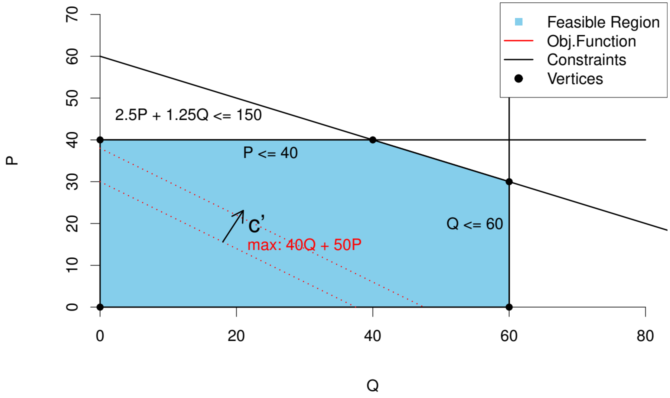
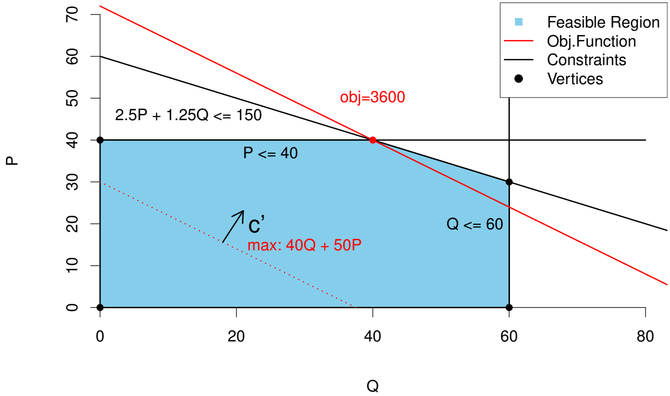
Graphical Representation of the LP 1



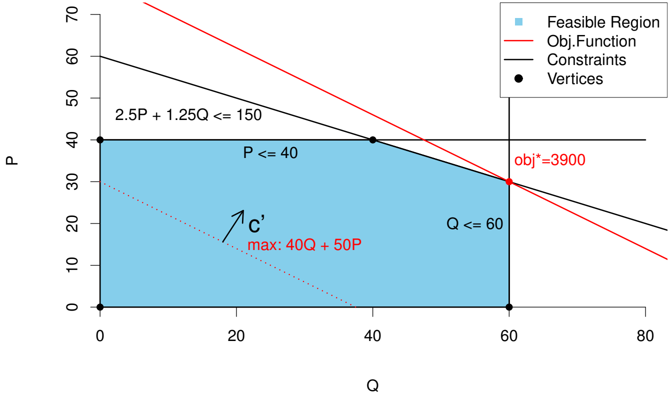
Graphical Representation of the LP 2



Graphical Representation of the LP 3



Graphical Representation of the LP 4



Algebraic modelling languages

Algebraic languages (like MathProg) allow to separate model, data und and algorithms

1. Development of a model
2. Provide data for model
3. Solve model
4. Analyze results

Commonly used languages:

- AMPL ("Basis" of MathProg)
- GAMS

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MathProg files

- Model in MathProg/GMPL format:
  filename.mod
- Results in GLPK format:
  filename.out
- Data for the model (optional):
  filename.dat

Structure

All in one file

statement;
statement;
. . .
statement;
`data;`
data block;
data block;
. . .
data block;
`end;`

Seperate model- and datafiles

Model file	Data file
statement;	`data;`
statement;	*data block*;
. . .	*data block*;
statement;	. . .
`end;`	*data block*;
	`end;`

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Structure

All in one file

statement;
statement;
. . .
statement;
`data;`
data block;
data block;
. . .
data block;
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Seperate model- and datafiles

Model file	Data file
statement;	`data;`
statement;	*data block*;
. . .	*data block*;
statement;	. . .
`end;`	*data block*;
	`end;`

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Parameters

param *name* [*alias*] [*domain*] , *attrib* , . . . , *attrib* ;

Attributes/Options:

`integer` specifies that the parameter is integer;

`binary` specifies that the parameter is binary;

`symbolic` specifies that the parameter is symbolic;

relation expression

(where *relation* is one of: <, <=, =, ==, >=, >, <>, !=)

:= *expression*

specifies a value assigned to the parameter;

`default` *expression*

specifies a value assigned to the parameter whenever no appropriate data are available in the data section.

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Variables

var *name* [*alias*] [*domain*] , [*attrib* , . . . , *attrib*] ;

Attributes/Options:

`integer` restricts the variable to be integer;

`binary` restricts the variable to be binary;

>= *expression*
specifies an lower bound of the variable;

<= *expression*
specifies an upper bound of the variable;

= *expression*
specifies a fixed value of the variable;

Constraints

Statement

s.t. *name* [*alias*] [*domain*] : *expression* , = *expression* ;

s.t. *name* [*alias*] [*domain*] : *expression* , <= *expression* ;

s.t. *name* [*alias*] [*domain*] : *express* , <= *express* , >= *express* ;

Examples:

```
subject to capacity cp:  x1 + x2 <= 10;
s.t. heat_constraint: Q_CHP + Q_B = Q_DEMAND;
```

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Objective Statement, Solve und Display

Objective

`minimize` *name* [*alias*] [*domain*] : *expression* ;

`maximize` *name* [*alias*] [*domain*] : *expression* ;

Example:

```
minimize obj: x + 1.5 * (y + z);
```

Solve & Display

`solve ;`

`display` [*domain*] : *item* , . . . , *item* ;

Objective Statement, Solve und Display

Objective

minimize *name* [*alias*] [*domain*] : *expression* ;

maximize *name* [*alias*] [*domain*] : *expression* ;

Example:

```
minimize obj: x + 1.5 * (y + z);
```

Solve & Display

solve ;

display [*domain*] : *item* , . . . , *item* ;

Short Example

```
param b := 0.6;
var x1;
var x2;
maximize obj: b * x1 + 0.5 * x2;
s.t. c1: x1 + 2 * x2 <= 1;
s.t. c2: 3 * x1 + x2 <= 2;
solve;
display x1, x2;
end;
```

Gusek IDE/ Commandline glpsol

IDE

`http://gusek.sourceforge.net/gusek.html`

Tools:

- Go → Solves the model (F5)
- Compile Model → Checks the model (Ctrl 7)
- Generate Outputfile on Go (yes/no)
- ...

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Dispatch Example

Dispatch Example - Overview

A reliable electrical power supply for a national economy with a given demand should be ensured over a time period of one year. For this purpose, different power generation technologies are available:

	Unit	Lignite	Coal	Gas
Installed capacity	GW_{el}	7	6	5
Fuel costs	EUR/GJ_{th}	1.8	3.4	7.2
Emission factor	t_{CO2}/TJ_{th}	100	95	56
Technology		Steam turbine (ST)	Steam turbine (ST)	Gas turbine (GT)
Net efficiency	%	41	47	39
Variable operational costs	EUR/MWh_{el}	6.1	4.0	2.0

CO_2 certificates amount to 12 EUR/t_{CO2}.

Excercise

Questions

- What would be the cheapest way to serve the (given) demand with respect to the fuel costs and variable operational costs?
- Develop a mathematical model by hand

	Unit	Lignite	Coal	Gas
Installed capacity	GW_{el}	7	6	5
Fuel costs	EUR/GJ_{th}	1.8	3.4	7.2
Technology		Steam turbine (ST)	Steam turbine (ST)	Gas turbine (GT)
Net efficiency	%	41	47	39
Variable operational costs	EUR/MWh_{el}	6.1	4.0	2.0

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LP Model

Abstract Formulation Using Sets

$$\text{min:} \quad \sum_{t \in T} \sum_{u \in U} (P_{t,u}(\frac{C_{fuel,u}}{\eta_u} + C_{var,u}))$$

$$\text{s.t.:} \quad \sum_{u \in U} P_{t,u} = D_t \quad \forall t \in T$$

$$P_{t,u} \leq P_{max,u} \quad \forall t \in T, \forall u \in U$$

$$P_{t,u} \geq 0 \quad \forall t \in T, \forall u \in U$$

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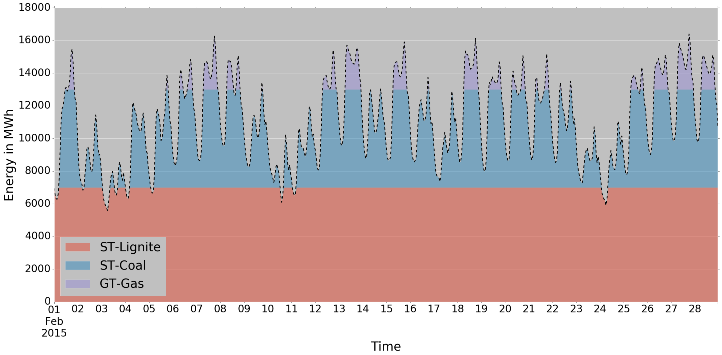
Dispatch Example (time-increment = 1 h)



What is the cost optimal way to dispatch a set of units?

What means optimal?

- From a mathematical point of view, we can (for some problems) proof optimality (LP, MILP)
- In the case of the energy system 'optimal' solutions should treated carefully:
  - Apply scenario and sensitivity analyses
  - Add stochastic programming to address uncertainty
  - Explore near-optimal solutions to think about effects of 'multiple optimal solutions' (particularly relevant for systems with generation of zero-marginal cost and storages)
  - Don't focus on the objective value but on the solution!

Irradiance Components

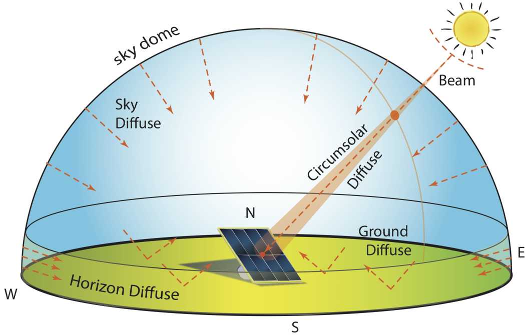


Figure: Irradiance components.
Source:https://www.e-education.psu.edu/eme810/node/683

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Sun Position

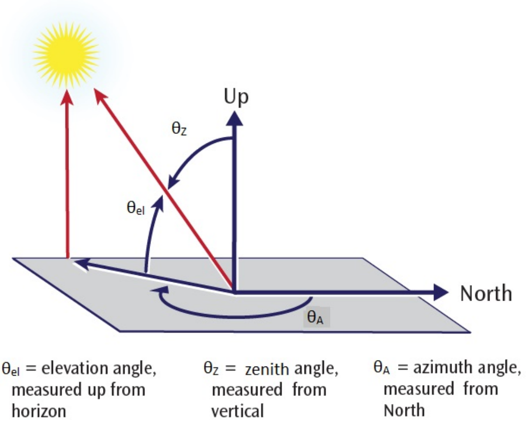

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Figure: Source: https://pvpmc.sandia.gov/modeling-steps/

Plane of Array (POA) Irradiance

A fundamental step in calculating PV performance is determining the irradiance incident on the plane of the array (POA) as a function of time. sThe POA irradiance is dependent upon several factors, including:

- Sun Position
- Array Orientation (fixed or tracking)
- Irradiance Components (Direct and Diffuse)
- Ground Surface Reflectivity (Albedo)
- Shading (near and far obstructions)

Equation for POA I

Consists of direct beam, groud reflected and sky diffuse :

$$E_{POA} = E_b + E_g + E_d$$

with:

$$E_b = DNI + \cos(AOI)$$

$$E_g = GHI \cdot albedo \cdot \frac{1 - \cos(\Theta_T)}{2}$$

$$E_d = \text{dependent on sky diffuse model}$$

Equation for POA II

Angle of incidence (AOI):

$$AOI = \cos^{-1}(\cos(\Theta_Z)\cos(\Theta_T) + \sin(\Theta_Z)\sin(\Theta_T)\cos(\Theta_A - \Theta_{A,array})$$

with:

$$\Theta_A := \text{solar azimuth}$$

$$\Theta_Z := \text{solar zenith}$$

$$\Theta_{A,array} := \text{array azimtuh}$$

$$\Theta_T := \text{array tilt}$$

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Module Temperature

The Sandia Temperature Model is defined as follows:

$$T_m = E_{POA} \cdot e^{a+b-WS} + T_a$$

with:

$$E_{POA} := \text{Irradiance on module in W/m2}$$

$$WS := \text{Windspeed (m/s)}$$

$$T_a := \text{Ambient Temperature in C}$$

The coefficients a and b are module specific (material, construction, mounting).

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Cell Temperature

Cell temperature is required to calculate the IV curve of the module. Cell temperature is affected by the incident irradiance, weather conditions (such as air temperature and wind speed), and module construction and material properties.
The Sandia Cell Temperature Model is defined as follows:

$$T_c = T_m + \frac{E_{POA}}{E_0} \cdot \Delta T$$

with:

$$E_0 := \text{Reference Irradiance with 1000 W/m2}$$

$$\Delta T := \text{Temperature Difference } T_c \text{ and } T_m \text{ at } E_0$$

Effective Irradiance

Effective irradiance is POA irradiance adjusted for angle of incidence losses, soiling, and spectral mismatch. In a general sense it can be thought of as the irradiance that is “available” to the PV array for power conversion.

Simplistic Approach with soiling factor SF (1 when clean):

$$E_e = \frac{E_{POA}}{E_0} \cdot SF \tag{6}$$

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Literature

- Information on the data sets: https://www.nrel.gov/docs/fy08osti/43156.pdf
- Equations and content is based on https://pvpmc.sandia.gov

Thank you, I hope there is time for questions...

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